Reply to the comment on 'Irreducible Green function theory for ferromagnets with first- and second-neighbour exchange'

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REPLY TO COMMENT

Reply to the comment on 'Irreducible Green function theory for ferromagnets with first- and second-neighbour exchange'

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Abstract. The consistency of the new irreducibility condition and that of the related irreducible Green function theory is discussed in the light of Brown's arguments.

Brown (1995) criticizes the new irreducibility condition used in our recent paper on irreducible Green function (IRG) theory (Mitra and Chakraborty 1995). He has been able to prove that the old irreducibility condition $\lambda = 0$ emerges from the new irreducibility condition which states that 'λ is independent of internal momentum index k''. We are confident that his proof is correct, but the conclusions drawn by him concerning the inconsistency and incorrectness of the new IRG theory seem to be wrong. Firstly, we would like to stress that the new condition does not imply that $A_{k'}$ is independent of k'. Along the lines of the arguments of Brown, we prove below that if the parameter $A_{k'}$ is chosen from the Callen scheme, λ vanishes also. We note that $\lambda_{k,k',q}$ can be written in the form

$$\lambda_{k,k',q} = \left\langle \left[F_{k,k',q} - 2\left(A_{k'} - A_{k-k'}\right) S_k^z \, \delta_{k-q} \right] \right\rangle \tag{1}$$

where

$$F_{k,k',q} = f_{k'} \, \delta_{k-k'-q} - f_{k-k'} \, \delta_{k'-q} \tag{2}$$

with

$$f_{k'} = 2\left(S_{k'}^z - \left\langle S_{k'}^z \right\rangle\right) S_{k-k'}^z + S_{k-k'}^- S_{k'}^+. \tag{3}$$

Using the Callen scheme for $A_{k'}$ we get

$$\begin{aligned} \lambda_{k,k',q} &= \left\langle \left[F_{k,k',q} - 2\alpha \left(\left\langle S_{k-k'}^- S_{k-k'}^+ \right\rangle - \left\langle S_{k'}^- S_{k'}^+ \right\rangle \right) S_k^z \, \delta_{k-q} \right] \right\rangle \\ \alpha &= \left\langle S^z \right\rangle / 2S^2. \end{aligned} \tag{4}$$

From (4) one can get readily

$$\lambda_{k,k'=0,q} = -\lambda_{k,k'=k,q} \tag{5}$$

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and, therefore,

$$\lambda_{k,k',q} = \lambda_{k,k'=0,q} = \lambda_{k,k'=k,q} = 0 \tag{6}$$

if λ is independent of k'.

It is worth noting that not only from the Callen scheme, but also any other form of $A_{k'}$ chosen symmetrically in spin operators, must yield the above result. Therefore, we may state that, instead of invalidating the new IRG theory, as Brown claims, his line of argument could actually be considered to provide us with a clear mathematical basis for the new irreducibility condition. Brown observes that if $A_{k'}$ is chosen in accordance with the Callen scheme it is independent of k', but we would like to point out that the form of $A_{k'}$ chosen in the new IRG theory depends on k'.

The above result is also certainly true for some general form of $A_{k'}$ such as

$$A_{k'} = C_1\left\langle S^-_{k-k'}S^+_{k-k'}\right\rangle + C_2\left\langle S^z_{k-k'}S^z_{k-k'}\right\rangle \tag{7}$$

where C_1 and C_2 are the coupling constants. It is to be noted that C_1 and C_2 cannot be found from the condition $\lambda = 0$, since even in the case of the above general form of $A_{k'}$, λ vanishes, irrespective of the values of C_1 and C_2. The above form of $A_{k'}$ reduces to that of Callen if we put $C_2 = 0$ and $C_1 = \alpha$. The inclusion of non-zero C_2 actually serves the purpose of accounting for more higher-order contributions to the zeroth-order diagrams. One may, however, argue that C_1 and C_2, being arbitrarily chosen parameters, prevent the IRG theory from being rigorous. We would like to assert that although C_1 and C_2 are arbitrarily chosen coupling constants, the accuracy in the choice is not important, since, whatever their magnitudes are, they contribute to the zeroth-order Green function, and the rest is accumulated in the self-energy part of the Dyson equation.

In conclusion, we emphasize that Brown's proof does not make the new IRG theory incorrect and inconsistent; rather, one finds that it traces the equivalence between the old and the new irreducibility conditions, and that our choice of the irreducible operator $\phi_{k,k',q}$ is such that any symmetric form of $A_{k'}$ serves to satisfy both the conditions.

References

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